P.R.GOVERNMENT COLLEGE (A), KAKINADA
B.Sc. THIRD YEAR MATHEMATICS - SEMESTER – VI (w.e.f. 2017-18)
Course: CLUSTER VIII- (A,B,D)-2 SPECIAL FUNCTIONS

No. Hours: 45hrs **Credits : 3**

Objectives:

- To understand the concepts of special functions which have applications in Physical Sciences.
- To learn finding power series solutions to some special types of differential equations.

UNIT-I HERMITE POLYNOMIAL: **(9 hrs)**

Hermite Differential Equations, Solution of Hermite Equation, Hermite's Polynomials, Generating function, Other forms for Hermite Polynomial, To find first few Hermite Polynomials, Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials. CHAPTER: 6.1 to 6.8

UNIT-II LAGUERRE POLYNOMIALS: **(9 hrs)**

Laguerre's Differential equation, Solution of Laguerre's equation, Laguerre Polynomials, Generating function, Other forms for the Laguerre Poynomials, To find first few Laguerre Polynomials,Orthogonal property of the Laguerre Polynomials, Recurrence formula for Laguerre Polynomials, Associated Laguerre Equation. CHAPTER: 7.1 to 7.9

UNIT-III LEGENDRE'S EQUATION: **(9 hrs)**

Definition, Solution of Legendre's Equation, Definition of $P_n(x)$ and $Q_n(x)$. General solution of Legendre's Equation(derivationis not required)To show that $P_n(x)$ is the coefficient of h^n in the expansion of $(1-2xh+h^2)^{1/2}$, Orthogonal properties of Legendre's Equation, Recurrence formulae, Rodrigues formula, CHAPTER: 2.1 to,2.8,2.12,

UNIT-IV BESSEL'S EQUATION: **(9 hrs)**

Definition, Solution of Bessel's General Differential Equations, General solution of Bessel's Equation, Integration of Bessel's equation in series for n=0,Definition of , $J_n(x)$ Recurrence formulae for $J_n(x)$, Generating function for $J_n(x)$ CHAPTER: 5.1 to 5.7

UNIT-V BETA AND GAMMA FUNCTIONS: **(9 hrs)**

Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions,Another form of Beta Function, Relation between Beta and Gamma Functions, Other Transformations. CHAPTER: 2.9 to 2.15

Prescribed text book: Special Functions by J.N.Sharma and Dr.R.K.Gupta.

BLUE PRINT FOR QUESTION PAPER PATTERN,

SEMESTER-VI, CLUSTER VIII –A, B,D -2

SPECIAL FUNCTIONS

UNIT	TOPIC	V.S.A.Q 1 M	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted
I	HERMIT POLYNOMIAL	01	02	01	19
II	LAGUERRE POLYNOMIAL	01	02	01	19
III	LEGENDRE'S EQUATION	02	02	01	20
IV	BESSEL'S EQUATION	02	02	01	20
V	BETA AND GAMA FUNCTIONS	02	02	02	28
Total		08	10	06	106

V.S.A.Q. = Very Short answer questions (1mark)
S.A.Q.= Short answer questions (5 marks)
E.Q .= Essay questions (8 marks)

Very Short answer questions : 8x1M = 08
Short answer questions : 6x5M =30
Essay questions : 4x8M = 32

Total Marks : 70

P.R. Government College (A), Kakinada
III B.Sc. Degree Examinations: Semester-VI, Mathematics
COURSE (Cluster VIII (A,B,D) 2) Special Functions

PAPER-VIII A,B, D 2 (MODEL PAPER w.e.f.2017-2018)

Time: 3 hours **Max. marks : 70M**

PART - I

Answer the following questions **8 x 1 = 8 M**

1. Write the generating function of Hermit's polynomial.
2. Show that $L_1(x) = 1 - x$.
3. Define Legendre's equation.
4. Show that $P_n(1) = 1$.
5. Define Bessel's equation.
6. Write $J_0(x)$.
7. Show that $\Gamma(1) = 1$.
8. Define Beta function.

PART - II

Answer any SIX questions by choosing three from each section. **6x5=30 M**

SECTION – A

9. Evaluate $\int_{-\infty}^{\infty} xe^{-x^2} H_n(x).H_m(x)$.
10. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.
11. Show that $L_2(x) = \frac{1}{2!}(2 - 4x + x^2)$
12. Show that $L_n(x) = \frac{e^x}{n!}\frac{d^n(x^n e^{-x})}{dx^n}$.
13. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.

SECTION – B

14. Show that $\int_{-1}^{1} P_m(x).P_n(x)\,dx = 0\ if\ m \neq n$.
15. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.
16. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.
17. Evaluate $\int_0^a x^4\sqrt{a^2 - x^2}$ dx.
18. Evaluate $\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}}\,dx$.

PART - III

Answer any FOUR questions by choosing at least one from each section. 6x5=30 M

SECTION – C

19. State and Prove Rodrigue's formula for $H_n(x)$.
20. Prove that $xL_n''(x) + (1-x)L_n'(x) + nL_n(x) = 0$
21. Prove that $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$.

SECTION –D

22. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.
23. When n is a positive integer, prove that $\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-1)^n 2^n \sqrt{\pi}}{1.3.3.....(2n-1)}$
24. Prove that $B(l,m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$

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