

**P.R. Government College (Autonomous), Kakinada**  
**III year B.Sc., Degree Examinations- V Semester (w.e.f 2018-19)**  
**Paper VI - Course: Linear algebra**

Total Hrs. of Teaching-Learning: 45 @ 3h / Week

Total Credits: 03

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**Objective:**

- To improve the students ability of understanding the most application oriented topic in Mathematics that is Linear Algebra.
  - To equip the skill of understanding the concepts and writing the proofs of the Theorems.
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**Unit I: Vector spaces – I**

(9 hrs)

Vector spaces, General properties of vector spaces, n-dimensional vectors, addition and scalar multiplication of vectors, internal and external composition, Null Space, Vector Subspaces, Algebra of subspaces, Linear sum of two subspaces, Linear combination of vectors, linear span, linear dependence and linear independence of Vectors.

**Unit II: Vector spaces – II**

(9 hrs)

Basis of vector space, Finite dimensional vector space, basis extension, coordinates, dimension of vector space, dimension of subspace, quotient space and Dimension of Quotient space.

**Unit III: Linear transformations**

(7 hrs)

Linear transformations, linear operators, properties of linear transformation, sum and product of linear transformations, Algebra of Linear Operators, Range space and NullSpace of LT, Rank and Nullity of a LT, Rank & Nullity theorem.

**Unit IV: Matrix**

(9 hrs)

Linear Equations, Characteristic Values and Characteristic Vectors of square matrix - Cayley-Hamilton Theorem.

**Unit V: Inner Product space:**

(11 hrs)

Inner Product spaces, Euclidean and Unitary spaces, Norm or length of a vector, Schwartz's inequality, Triangle Inequality, Parallelogram law, orthogonality and orthonormal set, complete orthonormal set, Gram-Schmidt Orthogonalisation Process, Bessel's inequality and Parsvels identity.

**Prescribed Text Books:**

J.N. Sharma & A.R.Vasista, Linear Agebra, Krishna Prakasham Mandir , Meerut.

**Books for Reference:**

1. III year Mathematics Linear Algebra and Vector Calculus, Telugu Academy.
2. A Text Book of B.Sc. Mathematics, Vol III, S.Chand&Co.

**QUESTION PAPER PATTERN, SEMESTER-V, PAPER –VI**

Unit	TOPIC	V.S. A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Vector spaces - I	02	01	01	15
II	Vector spaces - II	01	02	01	19
III	Linear Transformation	01	02	01	19
IV	Char. values and char. vectors	02	02	01	20
V	Inner product spaces	02	03	02	33
Total		08	10	06	106

V.S.A.Q. = Very Short answer questions (1mark)  
S.A.Q. = Short answer questions (5 marks)  
E.Q. = Essay questions (8 marks)

Very Short answer questions :  $8 \times 1M = 08$   
Short answer questions :  $6 \times 5M = 30$   
Essay questions :  $4 \times 8M = 32$

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Total Marks : 70

**P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA**  
**III YEAR B.Sc. DEGREE EXAMINATIONS – V SEMESTER**  
**MATHEMATICS COURSE: LINEAR ALGEBRA**  
**PAPER-VI (MODEL PAPER W. E. F. 2018-2019)**

**Time: 3 hours**

**Max. Marks: 70M**

**PART –I**

**Answer the following questions. Each question carries 1 mark.**

**8x1M = 8M**

1. Define linear combination of vectors
2. If  $S, T$  are the subspaces of vector space  $V(F)$  then  $L(S \cup T) = \dots$ .
3. The standard basis of  $V_2(R)$  is.....
4. Find the null space of the transformation  $T: R^2 \rightarrow R^3$  defined by  $T(x, y) = (x + y, x - y, y)$
5. Define Eigen vector of a square matrix.
6. Find the Eigen values of the matrix  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$
7. Find the length of the vector  $\alpha = (2, 1, 1 + i)$
8. Write Bessel's inequality.

**PART –II**

**Answer any Six questions by choosing Three from each section.**

**6 x 5M =30M**

**SECTION – A**

9. Determine whether the following set of vector is L.D or L.I  $\{(1,-2,1), (2,1,-1), (7,-4,1)\}$ .
10. Show that the set of vectors  $\{(2,1,4), (1,-1,2), (3,1,-2)\}$  form a basis for  $R^3$ .
11. If  $W$  is a subspace of a finite dimensional vector space  $V(F)$  then prove that  $W$  is also finite dimensional and  $\dim W \leq \dim V$
12. Find  $T(x, y, z)$  where  $T: R^3 \rightarrow R$  is defined by  $T(1,1,1)=3, T(0,1,-2)=1, T(0,0,1)=-2$ .
13. State and prove rank and nullity theorem.

**SECTION – B**

14. Solve the system of linear equations  $2x - 3y + z = 0, x + 2y - 3z = 0, 4x - y - 2z = 0$ .
15. State and prove Cayley - Hamilton theorem.
16. State and Prove Triangle inequality.
17. Prove that  $\left\{ \left( \frac{3}{5}, 0, \frac{4}{5} \right), \left( -\frac{4}{5}, 0, \frac{3}{5} \right), (0, 1, 0) \right\}$  form an orthogonal subset of  $\mathbb{R}^3(\mathbb{R})$  space.
18. State and prove Parseval's identity.

**PART –III**

**Answer any Four questions by choosing at least ONE from each section.                      4 X 8=32M**

**SECTION – C**

19. Let  $V(F)$  be a vector space and  $S = \{ \alpha_1, \alpha_2, \dots, \alpha_n \}$  is a finite subset of non-zero vectors of  $V(F)$ . Then  $S$  is linearly dependent if and only if some vector  $\alpha_k \in S, 2 \leq k \leq n$ , can be expressed as a linear combination of its preceding vectors.
20. Let  $W$  be a sub space of a finite dimensional vector space  $V(F)$ , then prove that  $\dim V/W = \dim V - \dim W$ .
21. Find the null space, range, rank and nullity of the transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  defined by  $T(x, y) = (x + y, x - y, y)$ .

**SECTION – D**

22. Find the characteristic roots and the corresponding vectors of the matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

23. State and Prove Cauchy-Schwarz's inequality.
24. Applying Gram Schmidt process obtain an orthonormal basis of  $\mathbb{R}^3(\mathbb{R})$  from the basis  $\{(2,0,1), (3,-1,5), (0,4,2)\}$ .

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**P.R.GOV.T.COLLEGE (AUTONOMOUS), KAKINADA**  
**III B.Sc. MATHEMATICS, Semester V (w.e.f 2016-2017)**

**Course Code: Linear Algebra**

**Total Hrs. of Exercises: 45 hrs @ 3 hr/Week in 15 Sessions**

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**Suggested topics for Problem Solving Sessions**

1. Vector Spaces
2. Basis and Dimensions-I
3. Basis and Dimensions-II
4. Linear Transformation
5. Linear Equations
6. Characteristic Values and **Cayley** Hamilton Theorem
7. Inner Product Spaces
8. Orthogonality

**PRACTICAL EXAMINATIONS PATTERN**  
**End of the V semester**  
**(Course: Linear Algebra)**  
**PRACTICAL EXAMINATION: 50 Marks**

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Written examination	: 25 M
Viva	: 05 M
Record	: 10 M
Cont.Ass.	: <u>10 M</u>
TOTAL	<u>50 M</u>