P.R.GOVT.COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS - Semester V (w.e.f 2018-2019)
Course: Ring Theory & Vector Calculus

Total Hrs. of Teaching-Learning: 45 @ 3 hr/Week **Total credits: 3**

Objectives:

- To impart knowledge on Ring Theory and its applications.
- To make awareness of the concepts of the transformation between curl Integration, Surface Integration and Volume integration.
- To Introduce the concepts of geometrical meaning of Gradient, Divergence and Curl.

RING THEORY

Unit – I: Rings – I **(11 hours)**

Definition of Ring and basic properties, Boolean Rings, divisors of zero and cancellation laws in Rings, Integral Domain, Division Ring and Fields, The characteristic of a ring – The characteristic of an Integral Domain, the characteristic of a Field, Sub rings and Ideals.

Unit – II: Rings – II **(11 hours)**

Definition of Homomorphism – Homomorphic Image – Elementary Properties of Homomorphism – Kernel of a Homomorphism – Fundamental Theorem of Homomorphism – Maximal Ideals – Prime Ideals.

VECTOR CALCULUS

UNIT – III: **Vector differentiation** **(9 hours)**

Vector differentiation –Ordinary Derivatives of Vector valued functions, Continuity and Differentiation, Gradient, Divergence, Curl operators, Formulae involving these operators.

UNIT – IV: Vector integration **(7 hours)**

Line Integral, Surface Integral, Volume Integrals with examples.

Unit – V: Vector Integration Applications **(7 hours)**

Gauss Divergence Theorem, Stokes theorem, Green's Theorem in plane and applications of these theorems**.**

Additional Inputs : Euclidean Ring definition and Examples.

Prescribed text Book:

A text book of Mathematics, Vol. III, S. Chand & Co.

Books for Reference:

1. Topics in Algebra by I.N.Herstine
2. Abstract Algebra by J. Fralieh, Published by Narosa Publishing house
3. Vector Calculus by Santhi Narayan, Published by S.Chand & Company Pvt. Ltd., New Delhi
4. Vector Calculus by R.Gupta, Published by Laxmi Publications.

QUESTION PAPER PATTERN, Semester-VI

Unit	**TOPIC**	**V.S.A.Q**	**S.A.Q**(including choice)	**E.Q**(including choice)	**Total Marks**
I	Rings – I	**02**	**03**	**01**	**25**
II	Rings – II	**02**	**02**	**O2**	**28**
III	Vector differentiation	**02**	**02**	**01**	**20**
IV	Vector integration	**01**	**02**	**01**	**19**
V	Vector Integration Applications	**01**	**01**	**01**	**14**
TOTAL		**08**	**10**	**06**	**106**

E.Q = Essay questions (8 marks)
S.A.Q = Short answer questions (5 marks)
V.S.A.Q = Very Short answer questions (1 mark)

Essay questions : 4x8M = 32
Short answer questions : 5x6M =30
Very Short answer questions : 8x1M =08

Total Marks : 70

P.R.GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III YEAR B.Sc., DEGREE EXAMINATIONS V SEMESTER
Mathematics: Ring Theory & Vector Calculus
Paper–V (Model Paper w. e. f. 2018-2019)

Time: 3 hours **Max. Marks : 70M**

Part –I

Answer all the following questions. 8x1M =8M

1. Define Boolean Ring.
2. Write the zero divisors of $(Z_9, +_9, X_9)$.
3. Find Kernel of the Homomorphism $f: Z(\sqrt{2}) \to Z(\sqrt{2})$ defined by $f(m+n\sqrt{2}) = m - n\sqrt{2} \ \forall\, m+n\sqrt{2} \in Z(\sqrt{2})$.
4. Give an example to show that every prime ideal need not be a maximal ideal.
5. Find div f , where $f = grad(x^3 + y^3 + z^3 - 3xyz)$
6. Evaluate $\int_0^1 (e^t \bar{i} + e^{-2t} \bar{j}) dt$.
7. State Green's theorem.
8. State the Green's Identities.

Part –II

Answer any THREE questions from each section. 6x5M =30M

Section – A

9. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.
10. Prove that the intersection of two ideals of a Ring R is an ideal of R.
11. Prove that a commutative ring R with unity having no proper ideals is a field.
12. Let R and R' be two rings and $f: R \to R'$ be a homomorphism. Then prove that the Kernel of f is an ideal of R.
13. Let C be the ring of Complex numbers and $M_2(R)$ be the ring of 2 x 2 matrices. If $f: C \to M_2(R)$ is defined by $f(a+ib) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ then prove that f is an into isomorphism and also find ker f.

Section – B

14. Find the directional derivative of $\phi = xy + yz + zx$ at A in the direction of $\overline{AB}$, where A = (1,2,-1), B= (-1,2,3).
15. Prove that div Curl $\bar{f} = 0$
16. If $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$, find the circulation of F round the curve , C where C is the Circle $x^2 + y^2 = 1, z = 0.$

17. Evaluate $\int_V F dV$ when $F = x\bar{i} + y\bar{j} + z\bar{k}$ and V is the region bounded by x=0, y=0, y=6, z=4 and $z=x^2$.

18. Evaluate $\oint_C (\cos x \,.\sin y - xy)\, dx + \sin x.\cos y\, dy$, by Green's theorem, where C is the circle $x^2 + y^2 = 1$.

Part –III

Answer any FOUR questions from the following choosing at least ONE question from each section. Each question carries 8 marks. 4X8M=32M

Section – C

1. Define the characteristic of a ring. Prove that the characteristic of an integral domain is either a prime or zero.
2. State and Prove fundamental theorem of homomorphism in rings.
3. Show that an ideal U of a commutative ring R with unity is maximal if and only if the quotient ring R/U is a field.

Section – D

22. Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla.A) - \nabla^2 A$
23. Evaluate $\int_S F.N\, dS$, where $F = zi + xj - 3y^2zk$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.
24. If $F = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ find $\int_S F.Nds$ by divergence theorem where S is surface of the cube bounded by x = 0,x=1 , y=0,y=1,z=0,z=1.

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P.R.GOVT.COLLEGE (AUTONOMOUS), KAKINADA

III B.Sc. MATHEMATICS, Semester VI (w.e.f 2016-2017)

Course Code: Ring Theory & Vector Calculus

Total Hrs. of Laboratory Exercises: 45 @ 3 hr / Week in 15 Sessions

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Suggested topics for Problem Solving Sessions

1. Rings and Characteristic of a Ring
2. Subrings and Ideals.
3. Homomorphism of a Ring
4. Directional Derivatives and Directional Derivative of Vector Point Function
5. Differential Operators
6. Integration of Vectors
7. Integral Transforms

Problem Solving Sessions Examinations Pattern
End of the VI semester
(Course: Linear Algebra, Multiple Integrals & Vector Calculus)

PRACTICAL EXAMINATION: 50 Marks

Written examination	: 25 M
Record	: 10 M
Viva- voce	: 05 M
Cont. Ass.	: 10 M
TOTAL	: 50 M