**P. R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA**
**II B.Sc. MATHEMATICS/Semester IV (w.e.f 2017-2018)**
**Course: Real Analysis**

**Total Hrs. of Teaching-Learning: 90 @ 6 hr/Week** **Total credits: 5**

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**OBJECTIVES:**

- Be able to understand and write clear mathematical statements and proofs.
- Be able to apply appropriate method for checking whether the given sequence or series is convergent.
- Be able to develop students ability to think and express themselves in a clear logical way.
- This curriculum gives an opportunity to learn about the derivatives of functions and its applications.

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**Unit 1: Real Number System and Real Sequence** **(18 hours)**

The algebraic and order properties of R – Absolute value and Real line – completeness property of R – applications of supreme property – intervals -Limit point of a set, Existence of limit points. (No questions to be set from this portion)

**Sequences and their limits** – Range and Boundedness of sequences - Necessary and sufficient condition for convergence of Monotone sequence, limit point of a sequence, Subsequences and the Bolzano Weierstrass theorem - Cauchy sequences – Cauchy's general principle of convergence theorems.

**Unit 2: Infinite Series** **(18 hours)**

Introduction to Infinite Series – convergence of series –Cauchy's general principle of convergence for series – Tests for convergence of nonnegative terms – p- test – limit comparision test – Cauchy's nth root test - De-Alambert's ratio test - alternating series – Liebnitz's test -absolute and conditional convergence.

**Unit 3: Limits and Continuity** **(18 hours)**

Real valued functions – Boundedness of a function - Limit of a Function, One-sided Limits- Right hand and Left Hand Limits - Limits at Infinity - Infinite Limits.(**no question to be set**)
Continuous Functions - Discontinuity of a Function - Algebra of Continuous Functions – Continuous functions on intervals - Some Properties of Continuity of a function at a point - Uniform Continuity**.**

**Unit 4: Differentiation and Mean Value Theorem** **(18 hours)**

The Derivability of a function, on an interval, at appoint, Derivability and Continuity of a function - Geometrical meaning of the Derivative - Mean Value Theorems - Rolle's Theorem, Lagranges Mean Value theorem, Cauchy Mean Value theorem.

**Unit 5: Riemann Integration** **(18 hours)**

Riemann sums, Upper and Lower Riemann integrals, Riemann integral, Riemann Integrable function – Darboux's Theorem - Necessary and sufficient conditions for Riemann integrability – properties of integrable functions – Fundamental Theorem of Integral Calculus – Integral as the limit of a sum – Mean Value Theorems.

**Additional Inputs :**

1. problems using cauchy's first theorem on limits and cauchy's second theorem on limits.
2. Statement of Maclaurin's theorem and expansions of $e^x, \sin x, \cos x, \log(1+x)$.

**Prescribed book:**

- Real Analysis by Rabert & Bartely and D.R.Sherbart, published by John Wiley.

**Reference books:**

- Elements of Real Analysis by Santhi Nararayan & M.D.Raisinghania, published by S.Chand& Company Pvt. Ltd., New Delhi.
- Course on Real analysis by N.P.Bali-Golden series publications
- A Text Book of Mathematics Semester IV by V.Venkateswarrao & others, published by S.Chand& Company Pvt. Ltd., New Delhi

**BLUE PRINT FOR QUESTION PAPER PATTERN**

**SEMESTER-III**

| TOPIC | Unit | V.S.A.Q | S.A.Q | E.Q | Marks allotted |
|---|---|---|---|---|---|
| **Real Number System and Real Sequence** | 1 | 1 | 1 | 1 | 14 |
| **Infinite Series** | 2 | 1 | 1 | 2 | 22 |
| **Limits and Continuity** | 3 | 1 | 1 | 1 | 14 |
| **Differentiation and Mean Value Theorem** | 4 | 1 | 1 | 2 | 22 |
| **Riemann Integration** | 5 | 1 | 1 | 2 | 22 |
| **TOTAL** | | 5 | 5 | 8 | 94 |

**V.S.A.Q =** Very short answer questions (1 mark)
**S.A.Q =** Short answer questions (5 marks)
**E.Q =** Essay questions (8 marks)

Very short answer questions : 5 X 1 = 05
Short answer questions : 3 X 5 = 15
Essay questions : 5X 8 = 40
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Total Marks = 60
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**P.R.GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA**
**II YEAR B.SC., DEGREE EXAMINATIONS IV SEMESTER**
**Mathematics: Real Analysis**
**Paper–IV (Model Paper w. e. f. 2018-2019)**

Time: 2 1/2 Hrs — Max. Marks: 60

## PART-I

**Answer ALL the questions. 5 X 1M = 5 M**

1. Define convergence of a sequence.
2. Test the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$.
3. Define continuity of a function at a point $'a'$.
4. Give an example of a function which is continuous but not derivable.
5. Evaluate $\lim_{n\to\infty} \frac{1}{n}[e^{\frac{3}{n}} + e^{\frac{6}{n}} + e^{\frac{9}{n}} + \cdots + e^{\frac{3n}{n}}]$.

## PART -II

**Answer any THREE questions each question carries FIVE marks. 3 X 5M = 15 M**

6. Show that $\lim_{n\to\infty}\left[\sqrt{\frac{1}{n^2+1}} + \sqrt{\frac{1}{n^2+2}} + \cdots + \sqrt{\frac{1}{n^2+n}}\right] = 1$.
7. State and Prove Leibnitz's test.
8. Examine for continuity the function $f$ defined by $f(x) = |x| + |x-1|$ at 0, 1.
9. Show that every derivable function on a closed interval is continuous.
10. State and prove fundamental theorem of Integral Calculus

## PART-III

**Answer any FIVE questions by choosing at least TWO from each section. 5 X 8M=40 M**

### SECTION –A

11. Prove that every monotonically increasing sequence which is bounded above converges to its least upper bound.
12. State and Prove Cauchy's $n^{th}$ root Test for the convergence of series.
13. Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - \sqrt{n^3}\right)$.
14. Prove that every continuous function is bounded and attains its bounds.

### SECTION-B

15. State and prove Rolle's theorem
16. Using LMVT prove that $1+x < e^x < 1+xe^x$, $\forall x > 0$
17. Prove that f(x) = sin x is integrable on [0, $\frac{\pi}{2}$] and $\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$
18. State the first mean value theorem of integral calculus and by using it prove that $\frac{\pi^3}{24} \le \int_0^{\pi} \frac{x^2}{5+3\cos x}\, dx \le \frac{\pi^3}{6}$.

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