P.R.GOVT.COLLEGE (AUTONOMOUS), KAKINADA
II B.SC. – MATHEMATICS / SEMESTER- III (W.E.F. 2017-2018)
Course: ABSTRACT ALGEBRA

Total Hrs. of Teaching: 90 @ 6 h / Week **Total Credits: 05**

Objective:

- **To learn about the basic structure in Algebra**
- **To understand the concepts and able to write the proofs to theorems**
- **To know about the applications of group theory in real world problems**

Unit 1: Groups **(20 hours)**

Binary Operation – Algebraic structure – semi group – monoid –Definition and elementary properties of a Group – Finite and Infinite groups – Examples – Order of a group – Composition tables with examples.

Unit 2: Subgroups, Cosets and Lagrange's Theorem **(20hours)**

Definition of Complex – Multiplication of two complexes – Inverse of a complex – Subgroup definition – examples - criterion for a complex to be a subgroup –criterion for the product of two subgroups to be a subgroup – union and intersection of subgroups.

Cosets definition – properties of cosets – Index of subgroup of a finite group – Lagrange's Theorem.

Unit 3: Normal Subgroups **(17 hours)**

Definition of normal subgroup – proper and improper normal subgroup – Hamilton group – criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – subgroup of index 2 is a normal subgroup – simple group – quotient group – criteria for the existence of a quotient group.

Unit 4: Homomorphism **(16 hours)**

Definition of homomorphism – Image of homomorphism – elementary properties of homomorphism – Definition and elementary properties of Isomorphism and automorphism – Kernel of a homomorphism – Fundamental theorem on homomorphism and applications.

Unit 5: Permutations and Cyclic groups **(17 hours)**

Definition of permutation – permutation multiplication – Inverse of a permutation – Cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

Definition of cyclic group - elementary properties – classification of cyclic groups.
Additional Inputs : Applications of group theory

Text Book:
Abstract Algebra by J.B.Fraleigh

Books for reference:

1 A text book of Mathematics, S.Chand and Co.
2. Modern Algebra by Gupta and Malik
3 Elements of Real Analysis by Santhi Nararayana & M.D.Raisinghania.

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-III

Unit	TOPIC	V.S.A.Q	S.A.Q	E.Q	Marks allotted to the Unit
1	Groups	1	1	2	22
2	Subgroups, Cosets & Lagrange's theorem	1	1	2	22
3	Normal Subgroups	1	1	1	14
4	Homomorphism	1	1	1	14
5	Permutations and Cyclic groups	1	1	2	22
		5	5	8	94

V.S.A.Q = Very short answer questions (1 mark)
S.A.Q = Short answer questions (5 marks)
E.Q = Essay questions (8 marks)

Very short answer questions : 5 X 1 =05
Short answer questions : 3 X 5 =15
Essay questions : 5 X 8 =40
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Total Marks = 60
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P.R.GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
II YEAR B.SC., DEGREE EXAMINATIONS III SEMESTER
Mathematics : Abstract Algebra
Paper–III : (Model Paper w.e.f. 2018 - 2019)

Time: 2 1/2 hours Max. Marks: 60M

Part–I

Answer the following questions. Each question carries 1 mark. 5x1M = 5 M

1. Write the Cauchy's composition table for $G = \{1, \omega, \omega^2\}$.
2. Write a proper subgroup of a group $G = \{1, -1, i, -i\}$ with respect to multiplication.
3. Define normal subgroup.
4. Check whether $f: (Z, +) \longrightarrow (Z, +)$ defined by $f(x) = x^2$ is a homomorphism or not.
5. Write the inverse of the permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 2 & 5 \end{pmatrix}$.

Part-II

Answer any Three questions. Each question carries FIVE marks. 5x5M=25 M

6. Prove that the set Z of all integers form an abelian group w.r.t. the operation defined by $a * b = a + b + 2 \; \forall \; a, b \in Z$.
7. Prove that a non empty complex H of a group G is a subgroup of G if and only if $H = H^{-1}$.
8. If M, N are two normal subgroups of G such that $M \cap N = \{e\}$ then every element of M commutes with every element of N.
9. If f is a homomorphism of a group G into a group G', then prove that the kernel of f is a normal subgroup of G.
10. Express the product $(2 \;\; 5 \;\; 4)(1 \;\; 4 \;\; 3)(2 \;\; 1)$ as a product of disjoint cycles and find its inverse.

Part-III

Answer any FIVE questions by choosing atleast two from each section. 5X8M=40 M

Section-A

11. Show that the n^{th} roots of unity form an abelian group with respect to multiplication.
12. Prove that a semi group $(G, .)$ is a group if and only if the equations $ax = b, ya = b \; \forall \; a, b \in G$ have unique solutions in G.
13. State and Prove the necessary and sufficient condition for a finite complex H of a group G to be a subgroup of G.

14. Prove that the union of two subgroups of a group is a subgroup if and only if one is contained in the other.

Section-B

15. If H is a normal subgroup of a group (G, .) then prove that the product of two right (left) cosets of H is also a right (left) coset of H.
16. Prove that every homomorphic image of a group G is isomorphic to some quotient group of G.
17. Prove that the product of two disjoint cycles defined on set commute.
18. Prove that every subgroup of a cyclic group is cyclic.

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