Operations Research-II

Game Theory

Games without Saddle point

Department of Statistics
PR Govt. College(A)
Kakinada

- Games without Saddle Point and solution:

If the games are probabilistic,, then there does not exist any saddle point.

i.e., Maximin‡Minimax

In this case, the games can be solved for optimum mixed strategies by the following methods

1. Algebraic method for 2x2 games

2. Dominance method for mxn games

3. Graphical method for 2xn and mx2 methods

4. method of sub games for 2x3 and 3x2 games

5. LPP method (general method)

➢ Algebraic method for 2x2 games:

Let us consider a 2x2 game,

a	**b**
c	d

the optimum mixed strategies for the players A and B and the game value can be obtained by the following formulae.

A1 -> $p1 = (d-c)/(a+d-b-c)$

A2 -> $p2 = 1-p$

B1 -> $q1 = (d-b)/(a+d-b-c)$

B2 -> $q2 = 1-q1$

value = $V = (ad-bc)/(a+d-b-c)$

➢ Algebraic method for 2x2 games:

Example: solve

3	**5**
8	2

Solution:

A1 -> p1 = (d-b)/(a+d-b-c)

= (2-8)/(2+3-5-8)=-6/-8 = 3/4

A2 -> p2 = 1-p = 1-3/4 = 1/4

B1 -> q1 = (d-b)/(a+d-b-c)

= (2-5)/-8 = -3/-8 = 3/8

B2 -> q2 = 1-q1 = 1-3/8 = 5/8

value = V = (ad-bc)/(a+d-b-c)

= (3x2-5x8)/-8 = -34/-8 = 4.25

A1	A2
3/4	1/4

B1	B2
3/8	5/8

V = 4.25

- **Dominance method for mxn games**:

The dominance method is used to reduce the mxn games to low dimension games(1x1,2x2,2x3,2x4,4x2,etc) in order to solve them by other appropriate methods, if possible.

The method has the following rules:

-> Rule 1:If the all elements of a row R1 ≥ the corresponding elements of another row R2, row R1 dominates R2.

we can delete R2 from the matrix.

-> Rule 2:If the all elements of a column C1 ≤ the corresponding elements of another column C2,

column C1 dominates C2.

we can delete C2 from the matrix.

-> Rule 3:If all the averages of two or more row elements ≥ the corresponding elements of another R’, the rows (combined) dominates R’.

we can delete R’.

Rule 3 can be applicable for columns also as in rule2.

➢ Dominance method:

Example: Solve

3	7	5
8	10	2
2	9	1

Solution:

Applying Rules of dominance:

3	7	5
8	10	2
2	9	1

clearly R2>R3, R3 can be deleted

3	7	5
8	10	2

clearly C1<C2, C3 can be deleted

Now, the game reduced to,

3	**5**
8	2

A1 -> p1 = (d-b)/(a+d-b-c)

= (2-8)/(2+3-5-8)=-6/-8 = 3/4

A2 -> p2 = 1-p = 1-3/4 = 1/4

B1 -> q1 = (d-b)/(a+d-b-c)

= (2-5)/-8 = -3/-8 = 3/8

B3 -> q2 = 1-q1 = 1-3/8 = 5/8

value = V = (ad-bc)/(a+d-b-c)

= (3x2-5x8)/-8 = -34/-8 = 4.25

A1	A2	A3
3/4	1/4	0

B1	B2	B3
3/8	0	5/8

V = 4.25

- **Graphical method for 2xn and mx2 games**:

2xn and mx2 games can be solved by graphical method. The method reduces 2xn and mx2 games to 2x2 game.

(1) Graphical method for 2xn games:

-> consider the 2xn game,

	B1	B2	----	Bn
A1	a11	a12	……	a1n
A2	a21	a22	……	a2n

steps:

-> Find the Expected payoffs of A corresponding to each of B as

B1 -> a11p1+a21p2

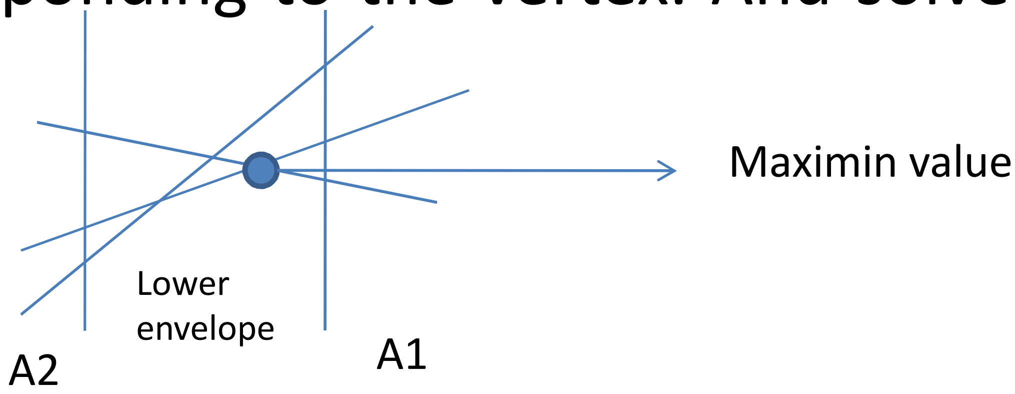
B2 -> a12p1+a22p2 and so on

-> Draw a graph with vertical axis for A1, A2 and plot the linear functions of expected payoffs on the graph as shown below.

-> Identify the lower region (lower envelope) of the graph as shown below.

-> Identify the vertices of the lower region and then find the vertex with high scale. It represents maximin value

-> form a 2x2 game with the strategies of B corresponding to the vertex. And solve the 2x2 game.



(2) Graphical method for mx2 games:

-> consider the mx2 game,

	B1	B2
A1	a11	a12
A2	a21	a22
….	…	….
Am	am1	am2

steps:

-> Find the Expected payoffs of B corresponding to each of A as

A1 -> $a_{11}q_1+a_{12}q_2$

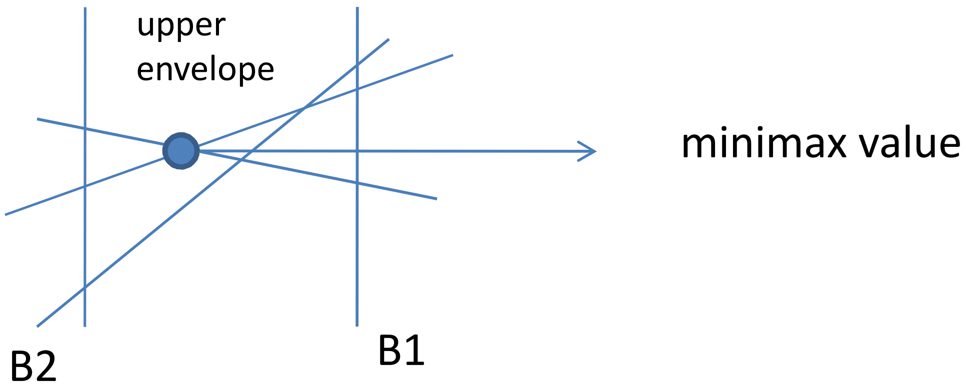
A2 -> $a_{21}q_1+a_{22}q_2$ and so on

-> Draw a graph with vertical axis for B1, B2 and plot the linear functions of expected payoffs on the graph as shown below.

-> Identify the upper region (upper envelope) of the graph as shown below.

-> Identify the vertices of the upper region and then find the vertex with low scale. It represents minimax value

-> form a 2x2 game with the strategies of A corresponding to the vertex. And solve the 2x2 game.



Example:

Solve the game by graphical method

	B1	B2	B3	B4
A1	2	5	8	10
A2	9	4	2	0

The Expected payoffs of A corresponding to each of B are

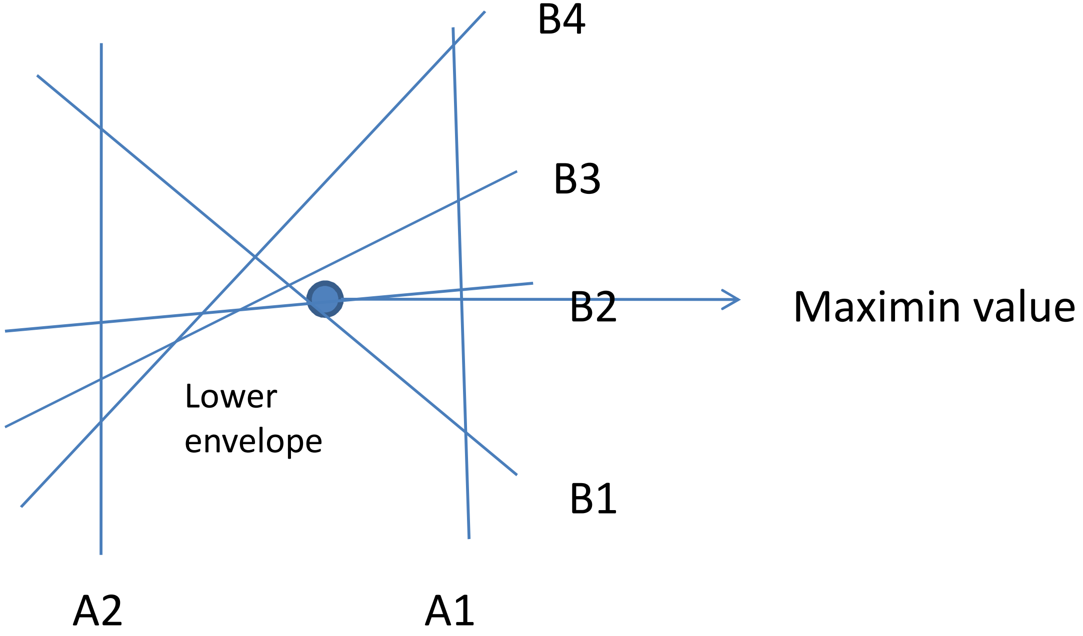
B1 -> $2p1+9p2$

B2 -> $5p1+4p2$

B3 -> $8p1 +2 p2$

B4 -> $10p1 +(0)p2$

Graph:



2x2 game is,

	B1	**B2**
A1	2	5
A2	9	4

solution is

A -> 5/8, 3/8

B -> 1/8, 7/8,0,0

V = 37/8

Self Assessment Questions:

1. Define a) Two-Person Zero-Sum games b) pure and mixed strategies c) payoff matrix d) saddle point
2. Explain maximin and minimax principle
3. Explain dominance method
4. Explain Graphical method to solve 2xn and mx2 games
5. Solve the games:

1.

4	-3
2	-6

2.

2	-3	5	6
2	-6	4	4
5	4	2	10

3.

2	5	10
14	10	3

4.

2	9
5	3
8	-4