Operations Research-I

Revised Simplex Method

Department of Statistics
P.R. Govt. College (A)
Kakinada

Revised Simplex Method –Algorithm

step1: Write the given LPP in Maximization standard form by introducing slack, surplus and artificial (if necessary) variables.

i.e., Max. $Z = CX$ sub. to $AX = b$ and $X \geq 0$

Step2: Begun with initial solution with basic matrix, $B = I_m$. i.e., $X_B = B^{-1}b = b$

step3: Insert the objective function as also one constraint as $Z - CX = 0$.

Now the system becomes, $A'X' = b_1$,

where $A' = \begin{pmatrix} A & 0 \\ -C & 1 \end{pmatrix}$, $X' = \begin{bmatrix} X \\ Z \end{bmatrix}$ and $b_1 = \begin{bmatrix} b \\ 0 \end{bmatrix}$

step4: Form the auxiliary basic, $B_1 = \begin{pmatrix} B & 0 \\ -C_B & 1 \end{pmatrix}$

and $B_1^{-1} = \begin{pmatrix} B^{-1} & 0 \\ C_B B^{-1} & 1 \end{pmatrix}$ and $A_1 = \begin{pmatrix} A_{NB} \\ -C_{NB} \end{pmatrix}$

Step5: Calculate the net evaluations $z_j - c_j$ by the formula, $z_j - c_j = (C_B B^{-1}, 1)\, A_1$

- If all $z_j - c_j \geq 0$, the current basic solution is an optimum basic feasible solution(OBFS).
- if at least one $z_j - c_j < 0$, find the most negative and enter the corresponding vector(say y_r) in to basis and go to step 6.

Step6: Calculate, $y_r = B_1^{-1}{}_{curr}. A_r$ and $X_{1B} = B_1^{-1}{}_{curr}\ b_1$

if all $y_r \leq 0$, then there exists an unbounded solution.

Otherwise , calculate ratios $\{x_{Bi}/y_{ir}, y_{ir}>0\}$ and find minimum and leave the vector (say y_k) from basis. And go to step 7.

Step7 : Write down the results from step2 to step6 in tabular form (revised simplex table).

Step8: Convert the leading element to unity and other elements of key column to zero by suitable row operations.

Step9: Go to step 4 and repeat the process until an OBFS is obtained or there exists an indication of unbounded solution.

Revised Simplex method Vs Simplex method:

Simplex Method

- The simplex method computes and stores more information that are not required at the current iteration and that are not relevance at subsequent iterations. So, it is not efficient for calculations on digital computer

- In this method for an mXn coefficient matrix, (m+1)X(n+2) elements need to written in table.

- Since there are large multiplications at each stage, and When working on digital computer, there may a problem with rounding off error.

Revised Simplex Method

- The revised simplex method computes and stores only necessary information that are required at the current iteration. So, it is more efficient for calculations on digital computer

- In this method for an mXn coefficient matrix, (m+1)X(m+3) elements only need to written in table.

- Since there are few multiplications at each stage, and When working on digital computer, it reduces the problem of rounding off error.

Revised Simplex Method –Problem:

Solve the LPP Max $Z = 2x_1+x_2$ STC $3x_1+4x_2\leq 6$, $6x_1+x_2\leq 3$ and x_1, $x_2\geq 0$ using revised simplex method.

Sol: Step1: By introducing slack variables $x_3\geq 0$, $x_4\geq 0$, the standard LPP is,

$$\text{Max } Z = CX \quad \text{STC } AX = b \text{ , } X \geq 0$$

where, $A = \begin{pmatrix} 3 & 4 & 1 & 0 \\ 6 & 1 & 0 & 1 \end{pmatrix}$ $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ $b = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

$C = (2 \quad 1 \quad 0 \quad 0)$

Step2: Let us choose $B = I_2$, initial solution is,

$x3 = 6, x4 = 3$ and $Z = 0$

Step3: Inserting the constraint $Z - CX = 0$,

$$A'X' = b_1$$

where $A' = \begin{pmatrix} 3 & 4 & 1 & 0 & 0 \\ 6 & 1 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 & 1 \end{pmatrix}$ $X' = \begin{pmatrix} x1 \\ x2 \\ z \end{pmatrix}$ $b_1 = \begin{pmatrix} 6 \\ 3 \\ 0 \end{pmatrix}$

Initial iteration:

Step4 : $B_1{}^{-1}{}_{curr} = \begin{pmatrix} B^{-1} & 0 \\ C_B B^{-1} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$A_1 = \begin{pmatrix} A_{NB} \\ -C_{NB} \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 6 & 1 \\ -2 & -1 \end{pmatrix}$ $C_B = (0, 0)$

$X_{1B} = B_1{}^{-1}{}_{curr}.b_1 = (6,3,0)^T$

Step5: The net evaluations,

$$z_j - c_j = (C_B B^{-1}, 1)\, A_1$$

$$= (0, 0, 1) \begin{pmatrix} 3 & 4 \\ 6 & 1 \\ -2 & -1 \end{pmatrix} = (-2, -1)$$

since z_1-c_1 is most negative, y_1 enters the basis

Step6: $y_1 = B_1^{-1}{}_{curr}.\, a_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix}$

Min{ X_{1B}/y_1, $y_1>0$} = Min{6/3, 3/6} = ½ for the basic variable x_4, y_4 leaves the basis. So, y_{12}= 6 becomes leading element.

Step7: Initial revised simplex table:

y_B	B_1^{-1}			Y_1	X_{1B}
Y_3	1	0	0	3	6
Y_4	0	1	0	6	3
Z	0	0	1	-2	0

First Iteration:

Step8: converting leading element (6) in to unity and other elements to zero, then

Step4: $B_1^{-1}{}_{curr} = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/6 & 0 \\ 0 & 1/3 & 1 \end{pmatrix}$ $X_{1B} = B_1^{-1}{}_{curr.} b_1 = \begin{pmatrix} 9/2 \\ 1/2 \\ 1 \end{pmatrix}$

Here x_3, x_1 are basic

Step5: The net evaluations,

$$z_j - c_j = (C_B B^{-1}, 1)\, A_1$$

$$= (0, 1/3, 1) \begin{pmatrix} 4 & 0 \\ 1 & 1 \\ -1 & 0 \end{pmatrix} = (-2/3, 1/3)$$

since z_2-c_2 is most negative, y_2 enters the basis

Step6: $y2 = B_{1}{}^{-1}{}_{curr.}\, a_2 = 1 \begin{pmatrix} -1/2 & 0 & 4 \\ 0 & 1/6 & 0 \\ 0 & 1/3 & 1 \end{pmatrix} \begin{pmatrix} \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 7/2 \\ 1/6 \\ 2/3 \end{pmatrix}$

Min{ X_{1B}/y_2, $y_2>0$} = Min{(9/2)/(7/2), (1/2)/(1/6)}
= 9/7 for the basic variable x_3, y_3 leaves the basis.
So, y_{21}= 7/2 becomes leading element.

Step7: First revised simplex table:

y_B	B_1^{-1}			Y_1	X_{1B}
Y_2	1	-1/2	0	7/2	9/2
Y_1	0	1/6	0	1/6	1/2
Z	0	1/3	1	-2/3	1

Second(Final) Iteration:

Step8: converting leading element (7/2) in to unity and other elements to zero, then

Step4: $B_1^{-1}{}_{curr} = \begin{pmatrix} 2/7 & -1/7 & 0 \\ -1/21 & 4/21 & 0 \\ 4/21 & 5/21 & 1 \end{pmatrix}$ $X_{1B} = B_1^{-1}{}_{curr.} b_1 = \begin{pmatrix} 9/7 \\ 2/7 \\ 13/7 \end{pmatrix}$

Here x_2, x_1 are basic

Step5: The net evaluations,

$$z_j - c_j = (C_B B^{-1}, 1)\, A_1$$

$$= (4/21, 5/21, 1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = (4/21, 5/21)$$

Since all z_j-c_j > 0, OBFS has been obtained.

The OBFS is,

$x_1 = 2/7$, $x_2 = 9/7$

and Max $Z = 13/7$

Self Assessment Questions:

1. Describe Revised Simplex Algorithm.
2. Write the advantages of revised simplex method over usual simplex method.
3. Solve the following LPP by revised simplex method

$$\text{Max } Z = 3X_1+5X_2,$$

$$\text{sub. to } \quad 2X_1+3X_2 \leq 6,$$

$$3X_1+2X_2 \leq 6 \quad \text{and } X_1, X_2 \geq 0.$$