- The equation of the sphere having (x_1, y_1, z_1) and (x_2, y_2, z_2) as the ends of the diameter is $(x - x_1)(x - x_2)+ (y - y_1)(y - y_2)+ (z - z_1)(z - z_2)$=0
- If the plane U=0 intersects the sphere S=0, in a circle C, then for all real values of $\lambda, S + \lambda U = 0$ represents the equation to a sphere passing through the circle C.
- If S=0, $S' = 0$ are two distinct intersecting spheres, then $\lambda_1 S + \lambda_2 S' = 0$ represents a system of spheres passing through the circle of intersection of the spheres S=0, $S' = 0$.

Problems

1. Find the equations of the spheres passing through the circle $x^2 + y^2$**=4**,Z=0 and is intersected by the plane x+2y+2z=0 in a circle of radius 3.

Sol: The equation of the sphere is $x^2 + y^2$ **−4+**λZ=0

Its centre is (0,0,-$\frac{\lambda}{2}$) $and\ the\ radius\ is$ $\sqrt{\frac{\lambda^2}{4} + 4}$

Given plane is x+2y+2z=0

Since perpendicular distance from the centre to the plane =radius of the sphere

$$\frac{\left|0+0+2*-\frac{\lambda}{2}\right|}{\sqrt{1+4+4}} = \sqrt{\frac{\lambda^2}{4} + 4}$$

By simplifying we get λ=$\pm$6

$\therefore$ *The equation of the sphere is* $x^2 + y^2$ **−4**$\pm$6Z=0

2. **Show that the two circles** $\boldsymbol{x^2+y^2+z^2-y+2z=0, x-y-z=2\ , x^2+y^2+z^2+x-3y+z-5=0, 2x-y+4z-1=0}$

Sol: The equation of the sphere is $x^2+y^2+z^2-y+2z+\alpha(x-y-z-2)=0(1)$

The equation of the sphere is $x^2+y^2+z^2+x-3y+z-5+\mu(2x-y+4z-1)=0$ (2)

Since both lie on the same sphere then

$$\alpha=2\mu+1, \alpha+1=3+\mu\ and\ \alpha+2=4\mu+1$$

By solving two of the equations we get $\alpha=3, \mu=1\ and\ the\ third\ eqation\ satisfied$

Therefore both circles lie on the same sphere

The equation of the sphere is by substituting $\alpha=3\ in\ (1) we\ get\ x^2+y^2+z^2$+3x-4y+5z-6=0

Angle of intersection of two spheres

- The two solids may intersect in different types. The type of intersection depends on their sizes, angle of intersection relative to their axes and relative position of their axes.
- The curve of intersection (COI) is a curve at the intersection of two or more solids. Curves of intersections are important for the production of components for engineering applications.

Angle of intersection of two spheres: The angle of intersection of two spheres is the angle between their tangent planes at the common point of intersection.

Theorem:

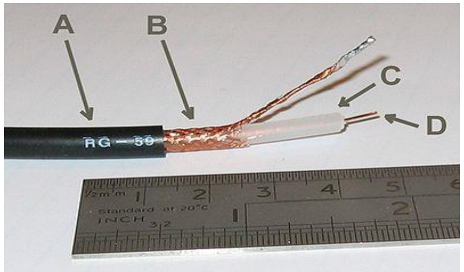
If r_1, r_2 are the radii, d is the distance between the centres, then angle of intersection is $\cos\theta = \frac{{r_1}^2+{r_2}^2-d^2}{2r_1r_2}$

If the spheres are orthogonal then, $r_1^2 + r_2^2 = d^2$

Co-axial system of spheres

two or more three-dimensional linear forms share a common axis is called coaxil system .

A coaxial cable, as a common example, is a three-dimensional linear structure.



Limiting Point:

Point spheres of a coaxial system of spheres are called limiting points of the system. Let$x^2 + y^2 + z^2 + 2\mu x + d = 0$ where d is a constant and μ is a paramenter, represent a coaxal system of spheres .

for any sphere of the system the radius is $\sqrt{\mu^2 - d}$ and centre is (- μ,0,0)

for limiting points of the system radius =0

So $\mu = \pm\sqrt{d}$

i) If d=0 and μ=0 and hence the system has only one imiting point (0,0,0). In this case the system s a tochig type of cosal system of spheres at (0,0,0)

ii) If d>0 then μ has two values $\pm\sqrt{d}$ and hence the system has two limiting points only

The limiting points are $(-\sqrt{d}, 0,0)$, $((\sqrt{d}, 0,0)$. In this case no two spheres of the system intersect.

iii) If d<0 the system has no limiting points. In this case the system is intersecting .

1. **Find the limiting points of the coaxal system defined by the spheres** $\boldsymbol{x^2+y^2+z^2+4\mathrm{x}+2\mathrm{y}+2\mathrm{z}+6=0}$ **and** $\boldsymbol{x^2+y^2+z^2+2\mathrm{x}-4\mathrm{y}-2\mathrm{z}+6=0}$

Sol:

Let S= $x^2+y^2+z^2+4\mathrm{x}+2\mathrm{y}+2\mathrm{z}+6=0$, $S'=x^2+y^2+z^2+2\mathrm{x}-4\mathrm{y}-2\mathrm{z}+6=0$

The radical plane of the sphere of coaxal system is S-S'=0

2x+2y+4z=0 i.e., x+y+2z=0

Equation of the sphere of the coaxal system of spheres is $x^2+y^2+z^2+4\mathrm{x}+2\mathrm{y}+2\mathrm{z}+6+\mu(\mathrm{x}+\mathrm{y}+2\mathrm{z})=0$

Centre is $\left(\frac{-(4+\mu)}{2},\frac{2-\mu}{2},-\mu-1\right)$ $radius\ is$ $\sqrt{\frac{(4+\mu)^2}{4}+\frac{(2-\mu)^2}{4}+(\mu+1)^2-6}$

For limiting points radius =0

By solving we get μ=0,-2

The limiting points are obtained by subtituting μ in centre (-2,1,-1),(-1,2,1)

Orthogonal spheres:-

The two spheres are said to be orthogonal to each other, if the tangent planes orthogonal to each other at the point of intersection.

Condition for Orthogonality:

Two spheres $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$, $x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=$ are said to be orthogonal if

$2u_1u_2+2v_1v_2+2w_1w_2=d_1+d_2$

Radical Plane

The radical plane of two spheres is defined as the locus of a point whose powers with respect to the spheres are equal.

Properties of the radical plane

- The radical plane of two spheres passes through their point of intersection.
- The radical plane of the two spheres is perpendicular to the line joining the centres.