Dielectrics

Differences between dielectrics and conductors.

S.No.	Dielectrics	Conductors
1.	Dielectric does not contain free electrons.	In conductors each atom contains at least one free electron.
2.	All the electrons are tightly bound to the nucleus of the atom.	The electrons in the outer most orbit are loosely bound by the nucleus.
3.	The conduction band is empty.	The conduction band contains electrons.
4.	The charge given to this is localized.	The charge given to this resides on the surface.
5.	The electrons take to and fro motion and do not leave the vicinity of the atom.	Here the electrons can take translatory motion and also leave the atom.
6.	Dielectrics do not conduct electricity.	Conductors conduct electricity.
7.	For a particular applied field strength, dielectric loses its insulation character, this minimum field strength is called break down strength.	There is no question of loosing conductivity character and break down strength does not arise.
8.	Examples :- Mica, Glass, Plastic etc.	Examples :- All metals.

Uses of dielectrics

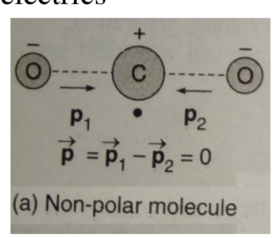
1. To increase the capacitance of the condensers dielectric materials like paper, Mica etc are placed in between the plates.
2. For insulation on electric conductors, dielectrics are used in form of layers around the conductors.
3. Dielectrics are also used for mechanical support to H.T. wires.
4. To increase the dielectric strength in electric fields dielectric materials are used.

Atomic view of dielectrics

The atoms or molecules consist of positive charges as well as negative charges in equal magnitude. The positive charges have one centre of gravity and negative charges have one centre of gravity.

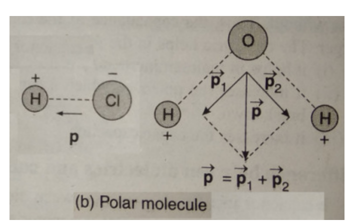
Dielectrics are of two types. 1) Non-polar dielectrics 2) Polar dielectrics

1) Non-polar dielectrics :- If the centre of gravity of positive charges coincides with the centre of gravity of negative charges, those molecules are called non-polar dielectrics.
Ex:- H_2, N_2, O_2, CO_2 etc.



(a) Non-polar molecule

2) Polar dielectrics :- If the centre of gravity of positive charges does not coincide with the centre of gravity of negative charges, those molecules are called polar dielectrics.
Ex:- H_2O, HCl, CO, N_2O etc.

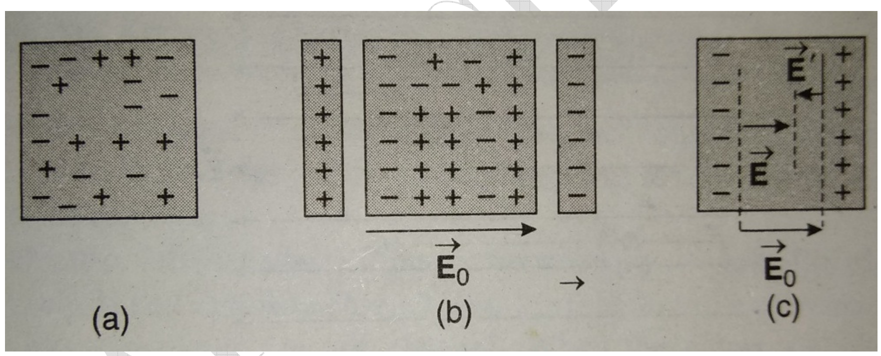


(b) Polar molecule

Behaviour of dielectrics in electric field

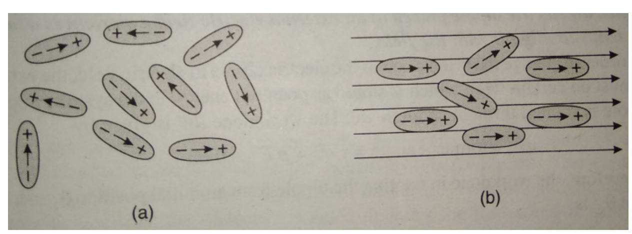
Non-polar dielectrics

- In general, non-polar dielectric molecules are randomly oriented such that the centres of gravity of positive and negative charges coincides with each other.
- When this material is placed between the electrodes of an electric field E_o, the molecules are reoriented such that the centres of gravity of positive charges are pulled towards the negative plate and vice versa. Then each molecule acts as a dipole.
- Separating the centres of gravity of positive and negative charges by applying electric field is called "polarization."
- If the applied field is increased then the separation also increases.
- Negative charges are induced on the surface of the dielectric which is towards the positive electrode and vice versa. These charges are called induced surface charges.
- The induced surface charges create induced electric field, "E^1" in opposite to the original electric field, E_o.
- The resultant electric field becomes $\boxed{E = E_o - E^1}$
- So, if the non-polar dielectric is placed in an electric field induced surface charges appear and the resultant electric field decreases.



Polar dielectrics

- In absence of electric field also the centres of gravity of positive and negative are separated in polar dielectrics. So each molecule acts as a dipole.
- These dipoles are randomly oriented but having some dipole moment called permanent dipole moment "p_p."
- If this material is placed in an electric field, the positive centres of dipoles are pulled towards the negative electrode of the field and negative centres of dipoles are pulled towards the positive electrode of the field. So, the alignment changes with an increase of dipole moment. This increased dipole moment is called induced dipole moment "p_i."



- The resultant dipole moment becomes $\boxed{p = p_p + p_i}$
- So, if the polar dielectric is placed in an electric field induced dipole moment arises and the resultant dipole moment increases.

Potential energy of a dipole in electric field

- Two equal and opposite charges separated by some distance is called electric dipole.
- Let the two charges be +q and –q and are separated by a distance '2a.'
- Then the dipole moment $p = 2aq$. The direction of dipole moment is from negative charge to positive charge.
- The dipole is placed in an electric field E. Let the dipole moment is making an angle θ with the electric field.
- Two equal and opposite forces F & F act on the charges +q and –q. These two forces constitute a couple.

The moment of couple or torque $\tau = Force\ x\ Perpendicular\ distance$

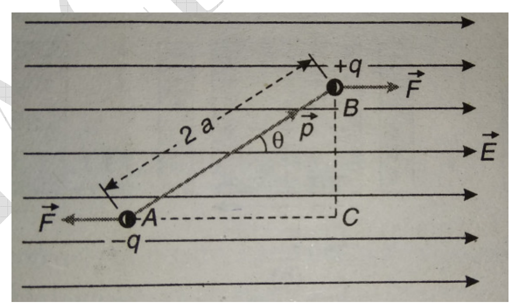
$$\tau = F\ x\ BC$$

But F = qE and from the Δ^{le} ABC in the figure $Sin\ \theta = \frac{BC}{AB}$ $(or)\ BC = AB\ Sin\ \theta$

$$\therefore\ \tau = qE.2a.Sin\ \theta \qquad \because AB = 2a$$

But 2aq = p = dipole moment $\therefore$ $\boxed{\tau = pE.Sin\ \theta}$

This can be written in the vector form as $\boxed{\bar{\tau} = \bar{p}\, x \bar{E}}$



This torque will rotate the dipole in the direction of the electric field.

Let dW be the work done in rotating the dipole through an angle dθ.

Then

$dW = Torque\ x\ angle\ of\ rotation\ = \tau\,.d\theta$

The total work done (W) in rotating the dipole from angle θ_1 to angle θ_2 is

$$W = \int dW = \int_{\theta_1}^{\theta_2} \tau\,.d\theta = \int_{\theta_1}^{\theta_2} pE\ Sin\ \theta\,.d\theta = pE \int_{\theta_1}^{\theta_2} Sin\ \theta\,.d\theta$$

$$W = -pE\,[Cos\ \theta]_{\theta_1}^{\theta_2} = -pE\,[Cos\theta_2 - Cos\ \theta_1]$$

<u>This work done is stored in the dipole as potential energy (U).</u>

$\therefore\ U = W = -pE\,[Cos\theta_2 - Cos\ \theta_1]$ (or) $\boxed{U = -pE\,[Cos\theta_2 - Cos\ \theta_1]}$

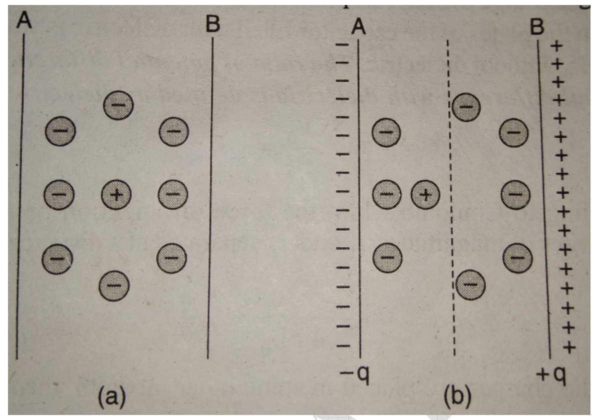
If the initial and final positions are $\theta_1 = 90^o$ and $\theta_2 = \theta$

Then U = - pE Cos θ (or) $\boxed{U = -\bar{p}.\bar{E}}$

i.	Case 1:- If $\theta = 0^o$ then U = - pE	the value is minimum.
ii.	Case 2:- If $\theta = 90^o$ then U = 0	the value is zero.
iii.	Case 3:- If $\theta = 180^o$ then U = + pE	the value is maximum.

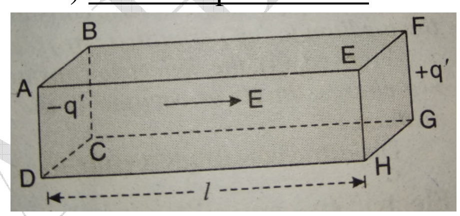
Dielectric polarization

- The atoms in dielectrics are symmetric when they are out side the electric field. Also the centres of gravity of positive and negative charges coincides each other.
- When the atoms are placed between plates of electric field, the electrons slightly displaced towards the positive plate and the nucleus slightly displaced towards the negative plate.



- The distorted atom is called electric dipole because the positive and negative charge centres are separated.
- Separating the centres of gravity of positive and negative charges by applying electric field is called "dielectric polarization."
- The dipole moment per unit volume is (the magnitude of) dielectric polarization.

Consider a dielectric slab having length l, and area of cross section A, placed in an electric field. Let the charges induced on the surfaces ABCD and EFGH of the dielectric are $-q^1$ and $+q^1$ respectively as shown in the figure.



$$Dielectric\ polarization\ (P) = \frac{Dipole\ moment}{Volume} = \frac{p}{V}$$

$$P = \frac{q^1 l}{Al} \qquad \because p = q^1 l \ \ and \qquad V = Al$$

$$\therefore \boxed{P = \frac{q^1}{A}}$$

Hence, *the dielectric polarization is equal to the induced surface charge density.*

Dielectric constant :-

Definition(1) :-It is defined as the ratio of the capacitance of a condenser with dielectric to the capacitance of the same condenser with out dielectric.

$$\boxed{Dielectric\ constant\ (K) = \frac{\text{Capacitance of the condenser with dielectric}}{\text{Capacitance of the condenser without dielectric}} = \frac{C}{C_o}}$$

We know that

Capacitance of the condenser without dielectric $C_o = \frac{\epsilon_o A}{d}$

Capacitance of the condenser with dielectric $C = \frac{\epsilon A}{d}$

Here ϵ_o = Permittivity of the free space or air. ϵ = Permittivity of the dielectric medium.

A = Area of the condenser plate. d = Distance between the plates of the condenser.

$$\therefore \quad K = \frac{\frac{\epsilon A}{d}}{\frac{\epsilon_o A}{d}} \quad i.e. \ \boxed{K = \frac{\epsilon}{\epsilon_o}} \quad \text{(or)} \quad \boxed{\epsilon = K\,\epsilon_o}$$

Definition(2) :- The dielectric constant of a dielectric medium is defined as the ratio of permittivity of the dielectric medium to the permittivity of the free space.

Also we know that the electrostatic forces between two charges in free space and in dielectric medium are given by $F_o = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2}$ & $F = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}$

From these two eqns. $\frac{F_o}{F} = \frac{\epsilon}{\epsilon_o} = K$

Definition(3) :- The dielectric constant of a dielectric medium is defined as the ratio of the electrostatic force between two charges in free space to the electrostatic force between the same two charges in dielectric medium.

Similarly we can write $\frac{V_o}{V} = \frac{E_o}{E} = \frac{\epsilon}{\epsilon_o} = K$

Definition(4) :- The dielectric constant of a dielectric medium is defined as the ratio of the potential difference between two points in free space to the potential difference between the same two points in dielectric medium.

Definition(5) :- The dielectric constant of a dielectric medium is defined as the ratio of the electric field intensity at a point in free space to the electric field intensity at the same point in dielectric medium.

Electric susceptibility :-

When a dielectric material is placed in an electric field that material is polarized. If the electric field (E) is increased then the polarization (P) also increases.

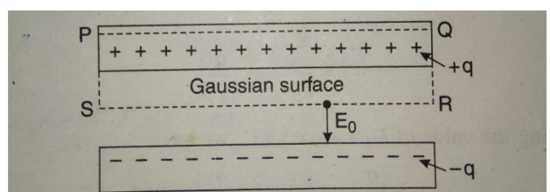
$\therefore \quad P \propto E \quad (or) \quad P = \chi E \quad (or) \quad \chi = \frac{P}{E}$

Here χ is a proportionality constant. It is called electric susceptibility.

Definition :- Electric susceptibility is the ratio of the electric polarization (P) created in the dielectric material to the electric field (E) applied.

Gauss's law in dielectrics

Consider a parallel plate capacitor with out dielectric (Figure.a). Let the charges on the two plates are +q and -q. The electric field between the plates is E_o.



(a) Parallel plate condenser without dielectric. *PQRS* is the Gaussian surface enclosing charge $+q$.

Then as per the statement of Gauss law

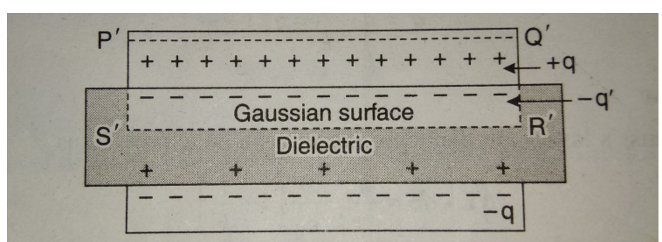
$$\oint \bar{E}_o \cdot d\bar{s} = \frac{q}{\epsilon_o}$$

Here q = charge with in the Gaussian surface PQRS.

$E_o \oint dS . = \frac{q}{\epsilon_o}$ (or) $E_o A = \frac{q}{\epsilon_o}$

Here A = Area of the plate.

$$\therefore \quad E_o = \frac{q}{\epsilon_o A} \longrightarrow (1)$$



(b) Parallel plate condenser with dielectric, *P′ Q′ R′ S′* is the Gaussian surface enclosing charge $(q - q')$

- ✪ A dielectric slab of dielectric constant K is placed in between the plates of the condenser.
- ✪ Then polarization takes place in dielectric and induced surface charge $-q^1$ is moved towards the positive plate of the condenser (figure-b).

- The net charge with in the Gaussian surface$P^1Q^1R^1S^1$ is $(q - q^1)$ and the electric field between the plates is E.

From Gauss law (with dielectric) $\oint \bar{E}\,.\,d\bar{s} = \frac{(q - q^1)}{\epsilon_o}$ ⟶ (2)

(Or) $EA = \frac{(q - q^1)}{\epsilon_o}$

$\therefore\ E = \frac{(q - q^1)}{\epsilon_o A}$ (or) $E = \frac{q}{\epsilon_o A} - \frac{q^1}{\epsilon_o A}$ ⟶ (3)

But $\frac{E_o}{E} = K$ (or) $E = \frac{E_o}{K}$ ⟶ (4)

Substituting eqn. (4) in (3) $\frac{E_o}{K} = \frac{q}{\epsilon_o A} - \frac{q^1}{\epsilon_o A}$ ⟶ (5)

Substituting eqn. (1) in (5) $\frac{q}{K\epsilon_o A} = \frac{q}{\epsilon_o A} - \frac{q^1}{\epsilon_o A}$ i.e. $\frac{q}{K} = q - q^1$

$\therefore\ q^1 = q - \frac{q}{K} = q(1 - \frac{1}{K})$ ⟶ (6)

Substituting eqn. (6) in (2) $\oint \bar{E}\,.\,d\bar{s} = \frac{1}{\epsilon_o}\,(q - q + \frac{q}{K})$

Or $\oint \bar{E}\,.\,d\bar{s} = \frac{1}{\epsilon_o}\,.\frac{q}{K}$ or $K \oint \bar{E}\,.\,d\bar{s} = \frac{1}{\epsilon_o}\,.\,q$

But $K\ \epsilon_o = \epsilon$

$$\therefore\ \boxed{\oint \bar{E}\,.\,d\bar{s} = \frac{1}{\epsilon}\,.\,q}$$

Statement :-The total normal electric flux over a closed surface in a dielectric medium is equal to $\frac{1}{\epsilon}$ times to the total charge with in the closed surface.

Electric displacement :- From the Gauss law in di-electrics

$K\,\epsilon_o \oint \bar{E}\,.\,d\bar{s} = q$ (or) $\oint \epsilon\bar{E}\,.\,d\bar{s} = q$

$$\boxed{\oint \bar{D}\,.\,d\bar{s} = q} \quad \because \bar{D} = \epsilon\,\bar{E}$$

Definition:- Electric displacement is an electric vector whose surface integral over a closed surface is equal to the free charge with in the closed surface.

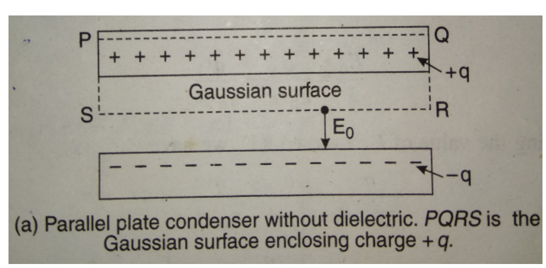
Three electric vectors and their relation (OR) Relation among $\bar{D}, \bar{E}$ & $\bar{P}$

Electric field intensity (E) :- The electric field intensity at a point in an electric field is equal to the electrostatic force acting on unit positive charge placed at that point.

Dielectric polarization (P) :- The dielectric polarization is the dipole moment per unit volume of the dielectric(or) It is the induced surface charge density $(P = q^1/A)$.

Electric displacement (D) :- Electric displacement is an electric vector whose surface integral over a closed surface is equal to the free charge with in the closed surface (or) It is the free surface charge density (D = q/A)

Consider a parallel plate capacitor with out dielectric **(Figure.a).** Let the charges on the two plates are +q and -q. The electric field between the plates is E_o.



(a) Parallel plate condenser without dielectric. *PQRS* is the Gaussian surface enclosing charge +*q*.

Then as per the statement of Gauss law

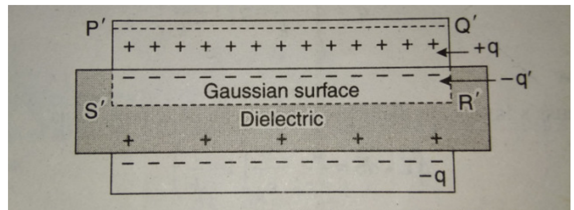
$$\oint \bar{E}_o \,.\, d\bar{s} = \frac{q}{\epsilon_o}$$

Here q = charge with in the Gaussian surface PQRS.

$$E_o \oint dS \,.= \frac{q}{\epsilon_o} \quad \text{(or)} \quad E_o A = \frac{q}{\epsilon_o}$$ Here A = Area of the plate.

$$\therefore \; E_o = \frac{q}{\epsilon_o A} \longrightarrow (1)$$

- A dielectric slab of dielectric constant K is placed in between the plates of the condenser.
- Then polarization takes place in dielectric and induced surface charge $-q^1$ is moved towards the positive plate of the condenser **(Figure-b).**
- The net charge with in the Gaussian surface$P^1Q^1R^1S^1$ is $(q - q^1)$ and the electric field between the plates is E.



(b) Parallel plate condenser with dielectric, $P'Q'R'S'$ is the Gaussian surface enclosing charge $(q - q')$

From Gauss law (with dielectric) $\oint \bar{E} \,.\, d\bar{s} = \frac{(q - q^1)}{\epsilon_o} \longrightarrow (2)$

$$\text{(Or)} \quad EA = \frac{(q - q^1)}{\epsilon_o}$$

$$\therefore \; E = \frac{(q - q^1)}{\epsilon_o A} \quad \text{(or)} \quad E = \frac{q}{\epsilon_o A} - \frac{q^1}{\epsilon_o A} \longrightarrow (3)$$

$$E = \frac{1}{\epsilon_o}\left(\frac{q}{A} - \frac{q^1}{A}\right) \quad \text{(or)} \quad \epsilon_o \, E = \left(\frac{q}{A} - \frac{q^1}{A}\right) \longrightarrow (4)$$

We know that

$$Induced\ surface\ charge\ density\ \left(\frac{q^1}{A}\right) = Dielectric\ polarization\ (P)$$

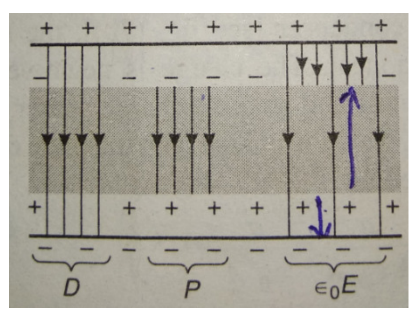
$$Free\ surface\ charge\ density\ \left(\frac{q}{A}\right) = Electric\ displacement\ (D)$$

Substituting these two values in eqn.(4)

$$\epsilon_o \, E = (D - P) \quad \text{(or)} \quad D = (\epsilon_o \, E + P)$$

D, E & P are vectors.

So, we can write this relation as $\boxed{\bar{D} = (\epsilon_o \, \bar{E} + \bar{P})}$



1) $\bar{D}$ is connected with the free charges only. It means its value can not change by the introduction of any dielectric.
2) $\bar{P}$ is connected with the induced charges in the dielectric only.
3) $\bar{E}$ is connected with free charges as well as induced charges.

Relation between dielectric constant and electric susceptibility

From the definitions

$$Dielectric\ constant\ (K) = \frac{Permitivity\ of\ the\ dielectric\ medium\ (\epsilon)}{Permitivity\ of\ free\ space\ (\epsilon_o)}$$

$$\boxed{K = \frac{\epsilon}{\epsilon_o}} \longrightarrow (1) \qquad \text{(or)} \qquad \epsilon = K\,\epsilon_o$$

& $$Electric\ susceptibility\ (\chi) = \frac{Electric\ polarization\ (P)}{Electric\ field\ intensity\ (E)}$$

$$\chi = \frac{P}{E} \quad \text{(or)} \quad P = \chi E$$

From the relation among D, E and P is $D = \epsilon_0 E + P$

$$\epsilon E = \epsilon_0 E + \chi E \qquad \because\ D = \epsilon E \ \ and \ \ P = \chi E$$

$$\epsilon = \epsilon_o + \chi \quad \text{(or)} \quad \boxed{\chi = \epsilon - \epsilon_o} \longrightarrow (2)$$

$$\chi = K\epsilon_o - \epsilon_o \quad \text{(or)} \quad \boxed{\chi = (K-1)\epsilon_o}$$

This is the relation between dielectric constant and electric susceptibility.

From eqns. (1) & (2) it is known that

1. The ratio of the permittivity of the dielectric medium to the permittivity of free space is dielectric constant.
2. The difference between the permittivity of the dielectric medium & the permittivity of free space is electric susceptibility.

Boundary conditions at the dielectric surface

The behavior of the electric displacement vector $\bar{D}$ and electric field vector $\bar{E}$ at the boundary between two dielectric media are called boundary conditions.

1. **Statement** :- The normal component of electric displacement vector is continuous across the charge free boundary between two dielectrics.

- Let AB be the boundary between two dielectric media 1 and 2. The two media are homogeneous (having same properties through out medium) and isotropic (having same properties in all directions).
- Consider a Gaussian surface in the form of a small pill box across the charge free boundary. Its height and the area of the curved surface is very small and negligible.
- Let ds be the area of the upper and lower surfaces .
- $\overline{D_1}$, $\overline{D_2}$ are the electric displacement vectors in media 1 & 2 on either sides of the boundary and are making angles θ_1 and θ_2 with the normal drawn to the boundary.

Applying Gauss law to the Gaussian surface (pill box)

$$\oint \bar{D}.\ d\bar{S} = q$$

$$\oint D cos\theta\ dS = q$$

$$D_1 cos\theta_1\ dS - D_2 cos\theta_2\ dS = q \longrightarrow (1)$$

But $D_1 cos\theta_1 = D_{1n}\ and\ D_2 cos\theta_2 = D_{2n}$

D_{1n} & D_{2n} are the normal components of displacement vectors on both sides of the boundary.

Then the eqn. (1) becomes $D_{1n}\ dS - D_{2n}\, dS = q \longrightarrow (2)$

But surface charge density $\sigma = \frac{q}{dS}$ (or) $q = \sigma\, dS$

Then $D_{1n}\ dS - D_{2n}\ dS = \sigma\ dS$

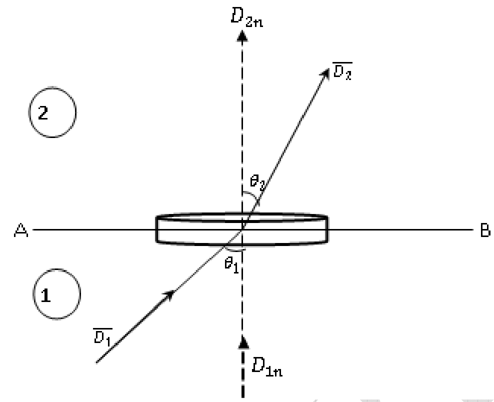
(or) $\boxed{D_{1n} - D_{2n} = \sigma}$

Since the boundary is charge free boundary $\sigma = 0$

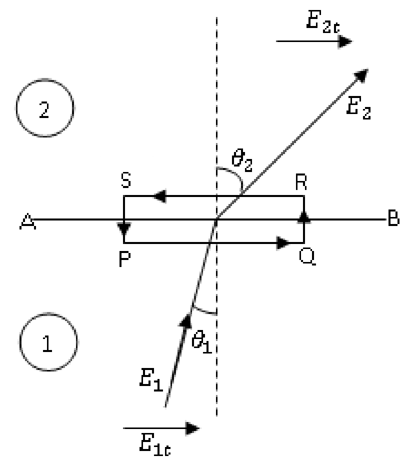
Then $D_{1n} - D_{2n} = 0$ (or)

$\boxed{D_{1n} = D_{2n}}$

So, the normal components of displacement vectors are equal on both sides of the charge free boundary, it means, the normal component of displacement vector is continuous across the charge free boundary.



2. **Statement** :- The tangential component of electric field vector is continuous across the boundary between two dielectrics.



- Let AB be the boundary between two dielectric media 1 and 2.
- Consider a path PQRS across the boundary AB. Its height QR & SP are very small and negligible and the length PQ = RS = dl.
- $\overline{E_1}$, $\overline{E_2}$ are the electric field vectors in media 1 & 2 on either sides of the boundary and are making angles θ_1 and θ_2 with the normal drawn to the boundary.
- The work done in moving unit positive charge around the path PQRSP is zero as the displacement is zero.

$$\oint \bar{E}.\ d\bar{l} = 0$$

$$\oint E sin\theta\ \ dl = 0$$

$$E_1 sin\theta_1\ (PQ) + \{-E_2 sin\theta_2\ (RS)\} = 0$$

$$E_1 sin\theta_1\ dl - E_2 sin\theta_2\ dl = 0 \longrightarrow (1)$$

But $E_1 sin\theta_1 = E_{1t}$ and $E_2 sin\theta_2 = E_{2t}$

E_{1t} & E_{2t} are the tangential components of electric field vectors on both sides of the boundary.

Then the eqn. (1) becomes $E_{1t}\ dl - E_{2t}\, dl = 0 \longrightarrow$ (2)

Then $\boxed{E_{1t} = E_{2t}}$

So, the tangential components of electric field vectors are equal on both sides of the boundary, it means, the tangential component of displacement vector is continuous across the boundary.

Courtesy:

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